

# Market Structure and the Distribution of Content

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# Motivation

- Entertainment products are ubiquitous.
  - Music, Movies, Books, etc.
- It is unclear how markets interact with defining product characteristics.
  - Both horizontal (genre) and vertical (quality) differentiation.
  - Quality is ex-ante uncertain.
  - Consumer demand is high, as is turnover.
- Digitalization has radically changed these markets.

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**How have changes in market structures influenced the distribution of products?**

Quality uncertainty leads to *allocative* inefficiency: producers inefficiently sort.  
Focus on three market structures:

- **Competitive Equilibria**
  - Inefficiently little product diversity.
- **Integrated Platform** (e.g., Netflix)
  - Coordinating production restores efficiency.
- **Recommendation System** (e.g., Spotify)
  - Can restore some efficiency by suppressing some content.

### Entertainment Industry

- Aguiar and Waldfogel (2018), Gambato (2025)

### Content Markets and Recommendation Systems

- Chen, Li, and Preuss (2025), Calvano et al. (2025)

### Contests

- Drugov and Ryvkin (2020), Ryvkin and Drugov (2020), Grossmann (2021)

# Model

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# Products and Producers

Mass  $m$  of ex-ante identical producers.

- Choose to produce in a genre  $g \in G = \{1, \dots, G\}$ .
- After this decision, quality  $v \sim F$  is realized (independently).

## Assumption

Let  $\bar{F}(v) = 1 - F(v)$ .  $F$  has increasing hazard rates:  $\frac{f(v)}{\bar{F}(v)}$  is strictly increasing.

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A strategy for producers corresponds to a mapping  $\Psi : [0, m] \rightarrow \Delta G$ .

- $\Psi$  induces a genre allocation  $\psi \in m \cdot \Delta G$  of producers:  $\psi_g = \int_0^m \Psi_g(i) di$ .
- $\psi$  induces  $\mu^\psi$  the measure of content:  $\mu_g^\psi((v, \infty)) = \psi_g \cdot \bar{F}(v)$ .

There is a unit mass of consumers.

- Each consumer has a type  $(g, t)$ :
  - $g$  determines which genre he will consume.
  - $t$  determines *how much* content he is willing to consume.
- Let  $T$  be the distribution of types.
- Let  $T_g$  be the distribution of demand for genre  $g$ , derived from  $T$ .

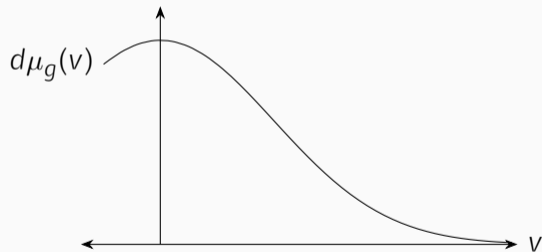
## Assumption

$T_g$  is regular, for each  $g$ :  $t - \frac{\bar{T}_g(t)}{d\bar{T}_g(t)}$  is increasing, and is full-support on  $[0, \bar{t}_g]$ .

## Consumer Utility without Prices

Consumers receive utility proportional to the quality of products they consume.

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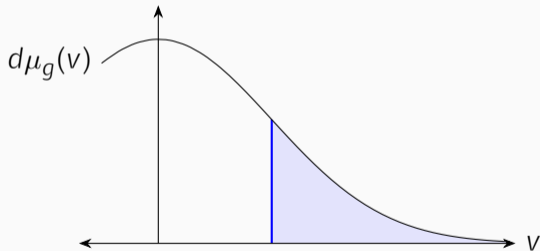
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Consumer  $(g, t)$ 's utility is

$$\int_{v_g(t)}^{\infty} v d\mu_g(v)$$

where  $v_g(t)$  is such that

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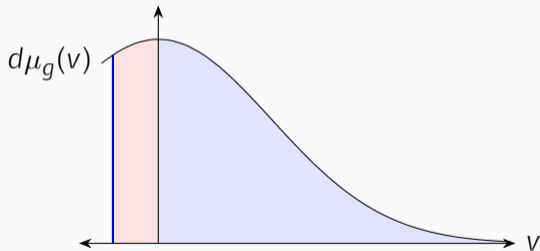
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# Prices

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For today, focus on linear pricing functions.

## Definition

I call a pricing function  $P$  linear if  $P(v) = p + \alpha(v - p)$  for  $p \in \mathbb{R}_+$  and  $\alpha \in [0, 1]$ .

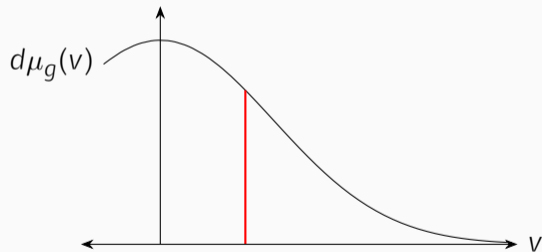
Denote this function  $P_{p,\alpha}$ .

- $p$  represents the base price.
- $\alpha$  represents the additional amount that producers can differentiate price based on quality.

## Consumer Utility with Prices

If the price of each good is  $P_{p,\alpha}(v) \neq 0$ , consumers will not consume to 0.

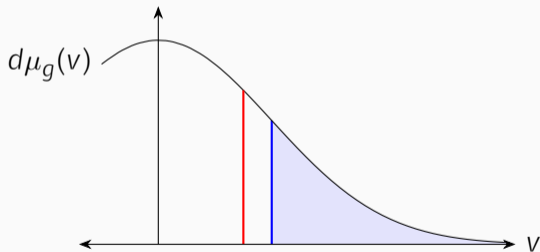
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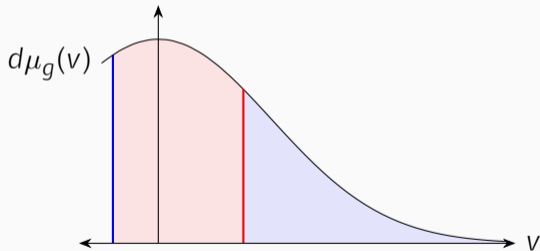
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## Producer Profits

Given consumer demand thresholds  $\{v_g(t)\}_{(g,t)}$ , expected profits from genre  $g$ :

$$\int_p^\infty \underbrace{\int_{v_g^{-1}(v)}^\infty P(v) dT_g(t)}_{\text{Profit from realization } v} f(v) dv = \int_0^\infty \underbrace{\int_{\max\{p, v_g(t)\}}^\infty P(v) f(v) dv}_{\text{Profit from consumer } (g,t)} dT_g(t)$$

- Producers choose genres to maximize expected profits
  - These choices induce  $\psi \in m \cdot \Delta G$ .
- Producer qualities are realized.
- Consumers choose bundles.
- Consumer utility and Producer profits are realized.

## Competitive Equilibrium

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## Producer Profits

Fix an allocation of producers  $\psi \in m \cdot \Delta G$  and  $(p, \alpha)$ .

For a consumer  $(g, t)$ , face content  $\mu_g^\psi$ .

- Threshold  $v_g(t)$  will solve  $t = \psi_g \bar{F}(v)$ .

Hence, will consume everything above  $\max \left\{ p, \bar{F}^{-1} \left( \frac{t}{\psi_g} \right) \right\}$ .

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Facing competition  $\psi$ , expected profits in genre  $g$  are given by

$$\int_0^\infty \Pi_{p,\alpha} \left( \min \left\{ \bar{F}(p), \frac{t}{\psi_g} \right\} \right) dT_g(t), \quad \text{where} \quad \Pi_{p,\alpha}(x) := \int_{\bar{F}^{-1}(x)}^\infty P_{p,\alpha}(v) f(v) dv$$

For  $(p, \alpha) = (0, 0)$ , let  $\Pi_{0,0}(x) := \int_{\bar{F}^{-1}(x)}^\infty f(v) dv$ .

# Competitive Equilibrium

## Definition

A competitive equilibrium  $\psi^{p,\alpha}$  for pricing function  $P_{p,\alpha}(v)$  is an allocation of producers such that

$$\psi_g > 0 \iff g \in \operatorname{argmax}_{g \in G} \int_0^\infty \Pi_{p,\alpha} \left( \min \left\{ \bar{F}(p), \frac{t}{\psi_g} \right\} \right) dT_g(t)$$

## Lemma

$\psi^{p,\alpha}$  is unique for each  $(p, \alpha)$ .

Let  $A^{p,\alpha} := \{g \mid \psi_g^{p,\alpha} > 0\}$  denote the set of “active” genres.

## The Integrated Platform

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## Integrated Platform: Setup

Consider a platform which controls production and sells to consumers.

- It doesn't charge per good, but is able to bundle.
- This allows the platform can extract full surplus from consumers.

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- This allows the platform can extract full surplus from consumers.

At an allocation of producers  $\psi$ , a consumer  $(g, t)$ 's utility from consumption is

$$\psi_g W\left(\min\left\{\bar{F}(0), \frac{t}{\psi_g}\right\}\right)$$

where

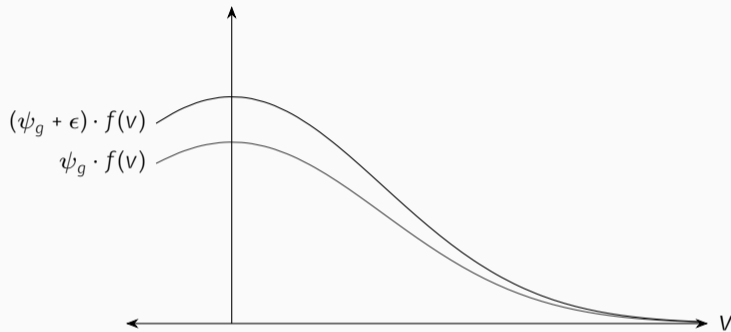
$$W(x) := \int_{\bar{F}^{-1}(x)}^{\infty} v f(v) dv$$

## Definition

An integrated optimal  $\psi^I$  is an allocation of producers such that

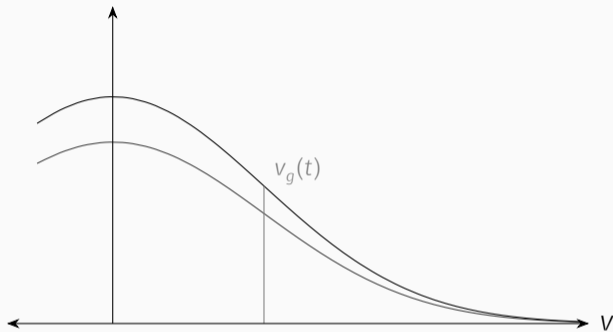
$$\psi^I \in \operatorname{argmax}_{\psi \in m\Delta G} \sum_{g \in G} \psi_g \cdot \int_0^\infty W\left(\min\left\{\bar{F}(0), \frac{t}{\psi_g}\right\}\right) dT_g(t)$$

## Solving the Platform's Problem



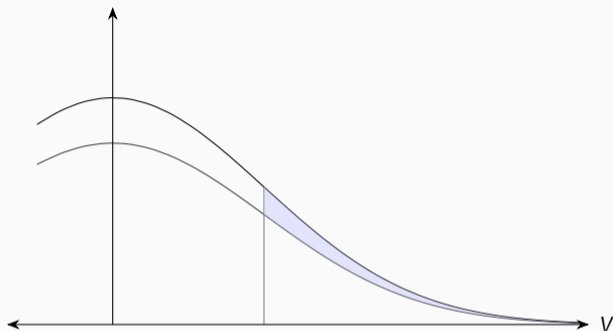
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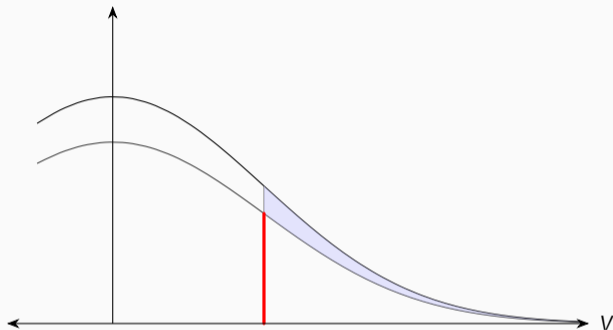
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$$\int_{v_g(t)}^{\infty} (v - \psi_g) f(v) dv$$

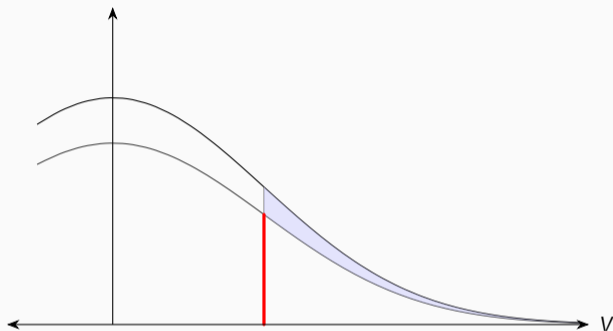
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Consider the benefit from increasing  $\psi_g$ :

$$M(x) := \int_{\bar{F}^{-1}(x)}^{\infty} \left( v - \bar{F}^{-1}(x) \right) f(v) dv$$

# Integrated Optimality

The platform hence sets

$$\int_0^{\infty} M\left(\min\left\{\bar{F}(0), \frac{t}{\psi_g}\right\}\right) dT_g(t)$$

equal across all active  $g$ .

## Lemma

The integrated optimal  $\psi^l$  is the unique allocation such that

$$\psi_g > 0 \iff g \in \operatorname{argmax}_{g \in G} \int_0^{\infty} M\left(\min\left\{\bar{F}(0), \frac{t}{\psi_g}\right\}\right) dT_g(t)$$

Let  $A^l := \{g | \psi_g^l > 0\}$ . This is the set of “active” genres.

# Market Failures and Quality Uncertainty

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## Two Possible Market Failures

“Posted-Price Problem”:

- In the competitive equilibrium, the base price  $p$  means that low-value goods are not consumed.
- By offering a menu of bundles, the platform is able to overcome this problem.

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Allocative Inefficiencies:

- How producers sort into different genres depends on the market structure.
- This sorting also depends crucially on the timing of quality realization.

## Ex-ante Quality Realization

An alternative is a setting where uncertainty is resolved before genre decisions. If producers know their quality they perfectly anticipate demand.

- The highest quality producers will claim the highest demand.

### Definition

- Let  $\mu^{p,\alpha}$  be the distribution of quality under the competitive equilibrium.
- Let  $\mu^I$  be the distribution of quality under the integrated platform.

They are both unique for producers with positive demand ( $v \geq p$  or  $v > 0$ ).

# Quality uncertainty is necessary and sufficient for allocative inefficiency

## Theorem

Fix any  $(p, \alpha)$ .

- (i) If quality is realized ex-ante, then there is no allocative inefficiency above the base price:  $\mu_g^{p, \alpha}(B) = \mu_g^l(B)$  for all  $g \in G$  and  $B \subseteq (p, \infty)$ .

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- (ii) If quality is realized ex-post, and the platform's allocation is non-degenerate ( $|A^l| > 1$ ), then there is generically allocative inefficiency:  $\psi^{p, \alpha} \neq \psi^l$ .

Genericity is with respect to  $T$ , and is required because of edge-cases (e.g., if  $T_g$  is equal for all  $g$ ).

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This allocative inefficiency is driven by the risk facing producers being different from the risk facing the platform.

## The Form of the Failure

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## Product Diversity

Recall that  $A^S$  is the set of active genres in market structure  $S$ :  $A^S := \{g : \psi_g^S > 0\}$ .

### Definition

Given two market structures,  $S, S'$ , I say that  $S$  features *less range* than  $S'$  if  $A^S \subseteq A^{S'}$ .

Product range is a coarse measure of concentration: how many genres are produced in a market structure.

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- (i) Range decreases in price differentiation:  $A^{p,\alpha}$  is shrinking in  $\alpha$ ;
- (ii) Range decreases in base price:  $A^{p,0}$  is shrinking in  $p$ ;
- (iii) All competitive equilibria have less range than the platform's optimal allocation:  $A^{p,\alpha} \subseteq A^I$ .

### Lemma

- (i) For all  $(p, \alpha)$ ,  $\Pi_{p,\alpha}$  is strictly increasing and concave. If  $\alpha = 0$ ,  $\Pi_{p,0}$  is linear.
- (ii)  $M$  is strictly increasing and strictly convex. (Due to increasing hazard rates.)

This implies that

- Price differentiation causes producers to weight low-demand consumers.
- The platform places low weight on low-demand consumers.
- When a genre is inactive, all consumers are effectively high-demand.

# Recommendation System

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Many platforms are not integrated; they do not directly control production.

- Their influence over producers comes from through recommendations to consumers.

These recommendation systems trade-off between

- Using recommendations to motivate producers; and
- Recommending the best content to consumers.

## Recommendation Mechanism

A mechanism is a set of possible “sub-genres” for producers. Sub-genres are

- A genre, specifying the type of content; and
- A recommendation function.

Producers still care about maximizing demand, and sort into sub-genres accordingly.

- Influencing producers requires suppressing content.

Formal Definition

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Producers still care about maximizing demand, and sort into sub-genres accordingly.

- Influencing producers requires suppressing content.

If the platform cannot commit, it implements the (0,0) competitive equilibrium.

Formal Definition

## Simplifying the Problem

The space of mechanisms is large, but optimal mechanisms are simple:

### Lemma

An optimal mechanism is  $p \in \mathbb{R}_+^G$  and  $\psi$  that maximize

$$\sum_{g \in G} \psi_g \cdot \int_0^\infty W\left(\min\left\{\bar{F}(p_g), \frac{t}{\psi_g}\right\}\right) dT_g(t)$$

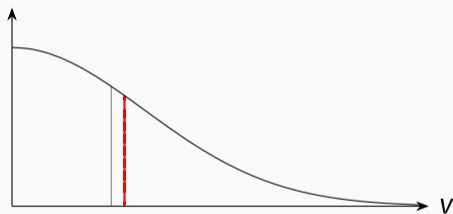
s.t.  $\psi_g > 0 \iff g \in \operatorname{argmax}_{g \in G} \int_0^\infty \min\left\{\bar{F}(p_g), \frac{t}{\psi_g}\right\} dT_g(t)$

Let  $\psi^R$  denote an optimal allocation.

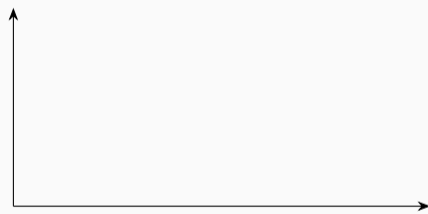
Formal Derivation

## Intuition: How can a mechanism help?

Consider a setting where  $A^{0,0} = \{1\}, A^I = \{1, 2\}$ .



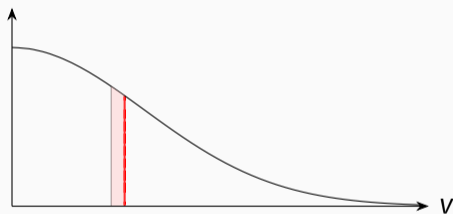
Genre 1



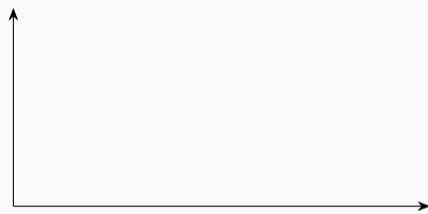
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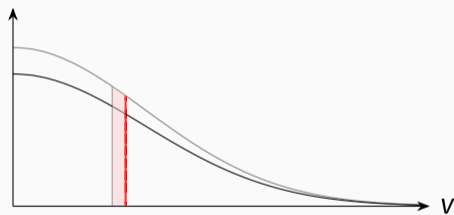
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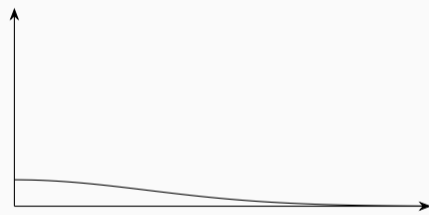
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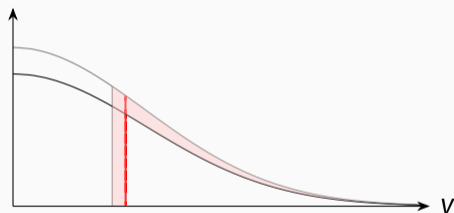
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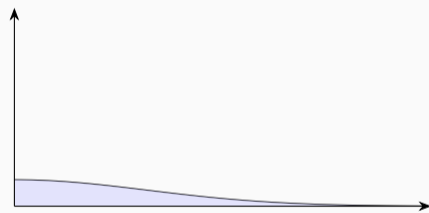
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Consider a setting where  $A^{0,0} = \{1\}, A^l = \{1, 2\}$ .



Genre 1



Genre 2

## Theorem

Fix an optimal mechanism and the corresponding active set  $A^R$ .

- (i) It is possible for the optimal mechanism to improve on the  $(0, 0)$  competitive equilibrium;
- (ii)  $A^{0,0} \subseteq A^R$ ; and
- (iii)  $A^R \subseteq A^I$ .

Both inclusions in (ii) and (iii) may be strict.

## Comparative Statics

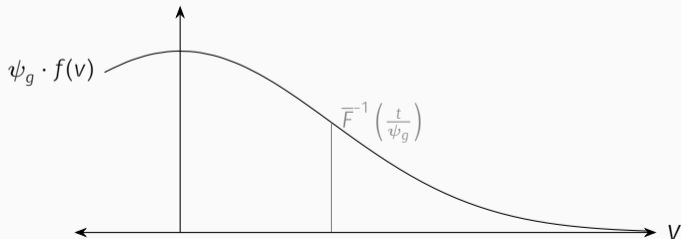
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# Consumer Heterogeneity

## Proposition

Consider two genres  $g, g'$  with  $T_{g'}$  a mean-preserving spread of  $T_g$ .

- (i)  $\psi_g^{p,\alpha} \geq \psi_{g'}^{p,\alpha}$ , for all  $p, \alpha$ .
- (ii) There exists an  $\bar{m}$  such that, for all  $m \leq \bar{m}$ ,  $\psi_g^l \geq \psi_{g'}^l$ , and for all  $m > \bar{m}$ ,  $\psi_g^l \leq \psi_{g'}^l$ .



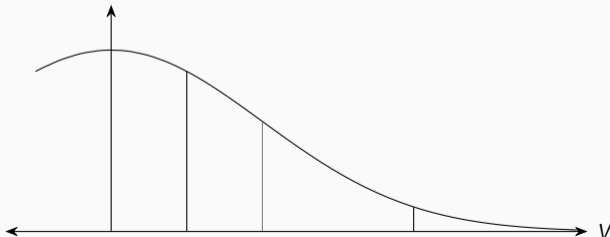
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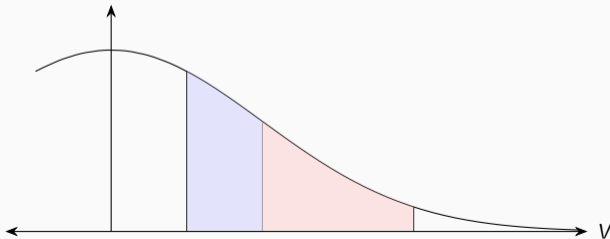
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## Proposition

Consider two genres  $g, g'$  with  $T_{g'}$  a mean-preserving spread of  $T_g$ .

(i)  $\psi_g^{p,\alpha} \geq \psi_{g'}^{p,\alpha}$ , for all  $p, \alpha$ .

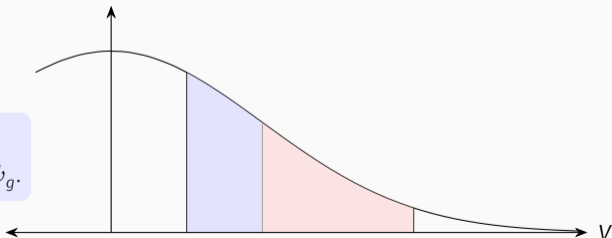
(ii) There exists an  $\bar{m}$  such that, for all  $m \leq \bar{m}$ ,  $\psi_g^l \geq \psi_{g'}^l$ , and for all  $m > \bar{m}$ ,  $\psi_g^l \leq \psi_{g'}^l$ .

Concavity of  $\Pi_{p,\alpha} \Rightarrow$

Producers respond by decreasing  $\psi_g$ .

Convexity of  $M \Rightarrow$

The platform responds by increasing  $\psi_g$ .



## Fatter tails lead to less range

Tightening the distribution encourages a wider distribution of content.

### Proposition

Fix a transformation  $v \mapsto \tau(v)$  where  $\tau$  is increasing with  $\tau(0) = 0$ . This induces distribution  $\hat{F}(v) = F(\tau^{-1}(v))$ .

- (i) If  $\tau$  is concave, then  $A^I \subseteq A^I_{\tau}$ .
- (ii) If  $\tau$  is convex, then  $A^I \supseteq A^I_{\tau}$ .
- (iii)  $A^{0,0} = A^{0,0}_{\tau}$ .

## Possible Directions and Conclusion

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## Heterogeneity in $F$ and Decreasing hazard rates

Introducing heterogeneity in  $F$  can create large disagreement between the competitive equilibrium and the socially optimal allocation.

- If pricing is uniform, firms are not sensitive to the differences in quality distributions. This leads them to over-invest in genres with “bad” distributions, but high demand.

### Decreasing Hazard Rates

- If  $F$  has *decreasing* hazard rates (very fat tails), there is too *much* product diversity in flat-price competitive equilibria.
- Corresponds to intuition from the contest literature.

## Other possible directions

Subscription pricing.

- Preliminary results: Genre-specific subscription pricing leads to too much product diversity.
- Would also be interesting to study the effect of bundling in these markets.

More general framework of concurrent contests.

- Effect of prize structures on endogenous sorting across contests.
- Also, producer effort and entry.

I build a model highlighting how ex-ante uncertainty over product quality leads to market inefficiencies.

- In particular, I show that it leads to excessive concentration.
- Introducing an integrated platform restores efficiency.
- Recommendation Systems are able to restore some efficiency, at the cost of suppressing some content.

## Definition

A mechanism is a set of sub-genres  $\mathcal{S}$  and an allocation  $\tilde{\psi} \in m\Delta\mathcal{S}$  such that

$$\sum_{g \in G} \int_0^\infty \left( \int_{\max\{v_g(t), 0\}}^\infty v d\mu_{g,t}(v) \right) dT_g(t)$$

such that

$$d\mu_{g,t}(v) = \sum_{s=(g,r_s)} \tilde{\psi}_s r_s(v,t) f(v)$$

$$t = \mu_{g,t}([v_g(t), \infty))$$

$$\tilde{\psi}_s > 0 \iff s \in \operatorname{argmax}_{s \in \mathcal{S}} \int_0^\infty \left( \int_{\max\{0, v_g(t)\}}^\infty r_s(v,t) f(v) dv \right) dT_g(t)$$

## Recommendation System's Problem

$$\max_{p_g, \psi_g, s_g, \bar{Q}} \sum_{x \in X} \psi_g \int W\left(\min\left\{\bar{F}(p_g), \frac{t}{\psi_g}\right\}\right) dT_g(t)$$

subject to the following constraints:

$$s_g + \int \min\left\{\bar{F}(p_g), \frac{t}{\psi_g}\right\} dT_g(t) - \bar{Q} = 0,$$

$$\psi_g s_g = 0,$$

$$m - \sum_{x \in G} \psi_g = 0,$$

$$\psi_g \geq 0, \quad s_g \geq 0, \quad p_g \geq 0$$

## Lemma

If a set of penalties  $p_g$  and producer allocations  $\psi_g$  are optimal, then there exist  $\bar{Q}$  and  $\eta$  such that for all  $g$  with  $\psi_g > 0$ :

$$\int \min \left\{ \bar{F}(p_g), \frac{t}{\psi_g} \right\} dT_g(t) = \bar{Q}$$

$$\int M \left( \min \left\{ \bar{F}(p_g), \frac{t}{\psi_g} \right\} \right) dT_g(t) + \bar{Q} \cdot p_g = \eta \quad \text{if } p_g > 0$$

$$\int M \left( \min \left\{ \bar{F}(0), \frac{t}{\psi_g} \right\} \right) dT_g(t) \geq \eta \quad \text{if } p_g = 0$$

$$\min p_g = 0$$

$$\sum \psi_g = m$$

If  $\psi_g = 0$ , it is without loss to set  $p_g = \infty$ .